TRIBHUVAN UNIVERSITY
INSTITUTE OF ENGINEERING
Examination Control Division
2081 Baishakh

Exam.	Back		
Level	BE	Full Marks	80
Programme	All(Except BAR)	Pass Marks	32
Year / Part	II / I	Time	3 hrs.

Subject: - Engineering Mathematics III *(SH 501)*

- ✓ Candidates are required to give their answers in their own words as far as practicable.
- ✓ *Attempt **All** questions.*
- ✓ *The figures in the margin indicate **Full Marks**.*
- ✓ *Assume suitable data if necessary.*

1. Use properties of determinant to prove: $\begin{vmatrix} (b+c)^2 & a^2 & a^2 \\ b^2 & (c+a)^2 & b^2 \\ c^2 & c^2 & (a+b)^2 \end{vmatrix} = 2abc(a+b+c)^3$ [5]

2. Prove that the necessary and sufficient condition for a square matrix A to possess an inverse is that A is non-singular. [5]

3. Define rank of a matrix. Reduce the matrix $\begin{bmatrix} 2 & -2 & 0 & 6 \\ 4 & 2 & 0 & 2 \\ 1 & -1 & 0 & 3 \\ 1 & -2 & 1 & 2 \end{bmatrix}$ into echelon form and hence find its rank. [1+4]

4. Find the modal matrix of the matrix $A = \begin{bmatrix} 1 & 0 & -1 \\ 1 & 2 & 1 \\ 2 & 2 & 3 \end{bmatrix}$ [5]

5. A vector field is given by $\vec{F} = \sin y\,\vec{i} + x(1+\cos y)\,\vec{j}$. Evaluate the line integral over the circular path given by $x^2 + y^2 = a^2, z = 0$ [5]

6. State Green's theorem. Using Green's theorem, find the area of asteroid $x^{\frac{2}{3}} + y^{\frac{2}{3}} = a^{\frac{2}{3}}$ [1+4]

7. State Gauss divergence theorem. Evaluate $\iint_S \vec{F}.\hat{n}ds$ for $\vec{F} = 2x\,\vec{i} + 3y\,\vec{j} + 4z\,\vec{k}$ where S is the surface of sphere $x^2 + y^2 + z^2 = 1$ by Gauss's divergence theorem. [1+4]

8. Use Stoke's theorem to evaluate $\iint_S \left(\nabla \times \vec{F}\right).\vec{n}\,ds$ where $\vec{F} = y\,\vec{i} + x(1-2z)\,\vec{j} - xy\,\vec{k}$ and S is the surface of the sphere $x^2+y^2+z^2=a^2$ above the xy-plane. [5]

9. Find the Fourier series of the function $f(x) = |x|$ for $-\pi \le x \le \pi$ in a Fourier series and deduce $\frac{\pi^2}{8} = \frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \ldots\ldots$ [4+1]

10. Find half-range Fourier sine series of $f(x) = e^{ax}$ in the interval $0 \le x \le \pi$. [5]

11. Discuss the existence of Laplace transform. Find the Laplace transform of [1+2+2]
(i) te^t (ii) sinhat sinat

12. Find the Inverse Laplace transform of a) $\frac{s^2}{(s-1)^3}$ b) $\frac{s^2}{(s^2+4)^2}$ [2+3]

13. Solve the following differential equation by Laplace transform method

$y''+3y'+2y = e^t, y(0) = 0 = y'(0)$ [5]

14. Solve the following linear programming problem using the big M method; [7]

Maximize $P = 2x_1+x_2$ subject to

$x_1+x_2 \leq 10$

$-x_1+x_2 \geq 2$

$x_1, x_2 \geq 0$

15. Use duality of simplex method to minimize, $z = 8x_1+9x_2$ subject to : [8]

$x_1+3x_2 \geq 4$

$2x_1+x_2 \geq 5$

$x_1, x_2 \geq 0$

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TRIBHUVAN UNIVERSITY
INSTITUTE OF ENGINEERING
Examination Control Division
2080 Bhadra

Exam.	Regular		
Level	BE	Full Marks	80
Programme	All (Except B. Arch)	Pass Marks	32
Year / Part	II / I	Time	3 hrs.

Subject: - Engineering Mathematics III *(SH 501)*

- ✓ Candidates are required to give their answers in their own words as far as practicable.
- ✓ *Attempt **All** questions.*
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1. Using the properties of determinant, prove the following identity. [5]

$$\begin{vmatrix} a & b & c \\ b+c & c+a & a+b \\ a^2 & b^2 & c^2 \end{vmatrix} = -(a-b)(b-c)(c-a)(a+b+c)$$

2. Define transpose of a matrix. Prove that every square matrix can be expressed as a sum of symmetric matrix and skew-symmetric matrix uniquely. [1+4]

3. Define rank of matrix. Find the rank of matrix $\begin{bmatrix} 3 & 1 & 4 \\ 0 & 5 & 8 \\ -3 & 4 & 4 \\ 1 & 2 & 4 \end{bmatrix}$ by reducing to normal form. [1+4]

4. State Cayley-Hamilton Theorem and verify it for the matrix. [1+4]

$$A = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix}$$

5. Prove that the integral $\int_A^B \vec{F}.d\vec{r}$ is independent of the path joining any two points A and B in a region if and only if $\int_c \vec{F}.d\vec{r} = 0$ for any simple closed curve C in the region formed by arcs joining the points A and B. [5]

6. State Green's theorem in plane and apply it to evaluate $\int_c [(2x - y + 4)dx + (5y + 3x - 6)dy]$ around a triangle in the xy-plane with vertices at (0,0), (3,0), (3,2). [1+4]

$y_3 = 2/9$ $s_1 = 29/9$

Mean $= 4/3$

7. State Gauss' divergence Theorem. Evaluate $\iint_S \vec{F}.\hat{n}\, ds$ where $\vec{F} = x\vec{i} + y\vec{j} + z\vec{k}$ and S is the surface of the sphere $x^2 + y^2 + z^2 = 1$ by using Gauss' divergence theorem. [1+4]

8. Evaluate $\int_C (xydx + xy^2dy)$ by stoke's theorem where C is the square in the xy-plane with vertices (1,1), (-1,1), (-1,-1) and (1,-1). [5]

9. Find the fourier series of the function $f(x) = x\sin x$ as a fourier series in $-\pi \leq x \leq \pi$. Also deduce that $\frac{1}{1.3} - \frac{1}{3.5} - \frac{1}{7.9} \ldots\ldots\ldots\ldots = \frac{\pi - 2}{4}$. [4+1]

10. Find half range Fourier sine series for $f(x) = x - x^2$ in $0 < x < 1$. [5]

11. State the condition for a function to exist its Laplace transform. Find the Laplace transform of : [1+2+2]

a) $\frac{1}{t}(e^{at} - \cos 6t)$

b) $f(t) = \begin{cases} \sin t, & \text{for } 0 < t < \pi \\ t & \text{for } t > \pi \end{cases}$

12. Find the inverse Laplace transform of [2+3]

a) $\tan^{-1}\left(\frac{1}{s}\right)$

b) $\frac{1}{s(s-4)^2}$

13. Solve the initial value problem by Laplace transform method: [5]

$y'' + y = \sin 3t;\ y(0) = 0,\ y'(0) = 0.$

14. Use the simplex method to solve the linear programming problem (constructing duality): [7]

Minimize $z = 3x_1 + 2x_2$ subject to $3x_1 - x_2 \geq -5$,

$-x_1 + 4x_2 \geq 1$, $x_1 + 9x_2 \geq 6$ and $x_1, x_2 \geq 0$.

15. Solve the linear programming problem by simplex method using Big-M method:

Maximize $F = 2x_1 + x_2$ subject to $2x_1 - x_2 \geq 2$,

$x_1 - x_2 \leq 2$, $x_1 + x_2 \leq 4$ and $x_1, x_2 \geq 0$. [8]

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TRIBHUVAN UNIVERSITY
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Examination Control Division
2080 Baishakh

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Year / Part	II / I	Time	3 hrs.

Subject: - Engineering Mathematics *(SH 501)*

- ✓ Candidates are required to give their answers in their own words as far as practicable.
- ✓ *Attempt __All__ questions.*
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1. Use properties of Determinant to show $\begin{vmatrix} a-b-c & 2a & 2a \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix} = (a+b+c)^3$. [5]

2. Show that for every complex square matrix, it can be uniquely express as the sum of a Hermitian matrix and a skew- Hermitian matrix. [5]

3. Test the consistency of the system of equations:
   $$x+2y-z=0,\ 2x+3y+z=10,\ 3x-y-7z=1.$$
   and solve completely if found consistent. [5]

4. Find the eigen-values of the matrix $A=\begin{bmatrix} 2 & -2 & 2 \\ 1 & 1 & 1 \\ 1 & 3 & -1 \end{bmatrix}$. [3+1+1]

   Hence use them to compute:

   a) $|A|$

   b) Eigenvalues of A^{-1}

5. Evaluate $\int_c \vec{F}.d\vec{r}$ where $\vec{F}=(\sin y)\vec{i}+x(1+\cos y)\vec{J}$ and the curve is circular path given by $x^2+y^2=a^2, z=0$. [5]

6. Evaluate $\iint_S \vec{F}.\vec{n}\,ds$ where $\vec{F}=y^2z^2\vec{i}+z^2x^2\vec{j}+x^2y^2\vec{k}$ and S is the surface of the sphere $x^2+y^2+z^2=1$ above the xy-plane. [5]

7. State Green's theorem in plane. Apply it to find the area of the curve $x^{2/3}+y^{2/3}=a^{2/3}$. [1+4]

8. Apply Gauss' divergence theorem to evaluate $\iint_S \vec{F}.\hat{n}\, ds$ where

$\vec{F} = (2xy+z)\,\vec{i} + y^2\,\vec{j} - (x+3y)\,\vec{k}$ and v is the region bounded by the surface of the planes 2x+2y+z = 6, x = 0, y = 0, z = 0. [5]

9. Obtain a Fourier series to represent $x + x^2$ for $-\pi \le x \le \pi$ and

Deduce that $\frac{\pi^2}{6} = \frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2}$ .... [5]

10. Define periodic function. Find the Fourier series of the function [2+3]

$f(x) = 2x - x^2$ in the interval (0,2)

11. State existence condition for Laplace transform. Obtain the Laplace transform of [1+2+2]

a) $\frac{1}{\sqrt{t}}$

b) $\frac{1-\cos 2t}{t}$

12. Obtain a Fourier series to represent x+x2 for $-\pi \le x \le \pi$ and

Deduce that $\frac{\pi^2}{6} = \frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2}$ .... [5]

13. Solve the initial value problem by Laplace transform method:

$y'' - 3y' + 2y = 4x + e^{3x};\quad y(0) = 1,\ y'(0) = -1.$ [5]

14. Solve the following Linear programming problem by using simplex method Max p = 15x+10y, subject to 2x+y ≤ 10, x+3y ≤ 10 and x, y ≥ 0. [7]

15. Solve the Linear Programming Problem by Big M method: [8]

Maximize: $P = 2x_1 + x_2 + 3x_3$

Subject to: $x_1 + x_2 + 2x_3 \le 5$

$2x_1 + 3x_2 + 4x_3 = 12$

and $x_1, x_2, x_3 \ge 0.$

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TRIBHUVAN UNIVERSITY
INSTITUTE OF ENGINEERING
Examination Control Division
2079 Bhadra

Exam.	Regular		
Level	BE	Full Marks	80
Programme	All (Except BAR)	Pass Marks	32
Year / Part	II / I	Time	3 hrs.

Subject: - Engineering Mathematics III *(SH 501)*

- ✓ Candidates are required to give their answers in their own words as far as practicable.
- ✓ *Attempt **All** questions.*
- ✓ *The figures in the margin indicate **Full Marks**.*
- ✓ *Assume suitable data if necessary.*

1. Prove that $\begin{vmatrix} a^3 & 3a^2 & 3a & 1 \\ a^2 & a^2+2a & 2a+1 & 1 \\ a & 2a+1 & a+2 & 1 \\ 1 & 3 & 3 & 1 \end{vmatrix} = (a-1)^6$ by using properties of determinate. [5]

2. Define transpose of a matrix. Prove that the transpose of the product of two matrices is the product of their transpose taken in reverse order. [1+4]

3. Find the rank of the matrix $\begin{bmatrix} 1 & 0 & -5 & 6 \\ 3 & -2 & 1 & 2 \\ 3 & -2 & -9 & 14 \\ 4 & -2 & -4 & 8 \end{bmatrix}$ by reducing it into normal form. [5]

4. State Cayley-Hamilton Theorem. Use it to find the inverse of the matrix: [1+4]

$$\begin{bmatrix} 1 & 1 & 3 \\ 1 & 3 & -3 \\ 2 & -4 & -4 \end{bmatrix}$$

5. Prove that the line integral $\int_C \vec{F}.d\vec{r}$ of a continuous vector function $\vec{F}$ defined in a region R is independent of the path C joining any two points in R if and only if there exists a single valued scalar function ϕ, having first order partial derivatives such that $\vec{F} = \nabla\phi$. [5]

6. Evaluate $\iint_S \vec{F}.\vec{n}\, ds$ where $\vec{F} = y^2z^2\vec{i} + z^2x^2\vec{j} + x^2y^2\vec{k}$ and S is the surface of the sphere $x^2 + y^2 + z^2 = 1$ above the xy-plane. [5]

7. Apply Green's theorem in plane to evaluate, $\int_C \vec{F}.d\vec{r}$ where $\vec{F} = (x^2 - xy^3)\vec{i} + (y^2 - 2xy)\vec{j}$ and C is a square with vertices (0, 0), (2, 0), (2, 2), (0, 2). [5]

8. Verify the stroke's theorem for $\vec{F} = (x^2 + y^2)\vec{i} - 2xy\vec{j}$ taken round the rectangle bounded by the lines $x = \pm a$, $y = 0$, $y = b$. [5]

9. Define Laplace transform of function f(t). Find the Laplace transform of [1+2+2]

a) $te^{-4t}\sin 3t$ b) $\frac{1-e^{t}}{t}$

10. Find the inverse Laplace transform of: [2+3]

a) $\frac{s^2}{(s+2)^3}$ b) $\tan^{-1}\frac{2}{s}$

11. Solve the following initial value problem by using Laplace transform

$y'' + 2y' - 3y = \sin t,\ y(0) = y'(0) = 0.$ [5]

12. Find the Fourier series of the function $f(x) = \frac{(\pi - x)^2}{4}$ in the interval $0 \le x \le 2\pi$. [5]

13. Obtain the half-range Fourier cosine series of sinx in the interval $0 \le x \le \pi$. [5]

14. Solve the linear programming problem maximize by simplex method [7]

Maximize: $Z = 10x_1 + x_2 + 2x_3$

Subject to: $x_1 + x_2 - 2x_3 \le 10$

$4x_1 + x_2 + x_3 \le 20$

and $x_1, x_2, x_3 \ge 0$.

15. Solve the linear programming problem by simplex method using two phase method: [8]

Maximize $Z = 3x_1 - x_2$

Subject to $2x_1 + x_2 \ge 2$

$x_1 + 3x_2 \le 2$

$x_2 \le 4;\ x_1, x_2 \ge 0$.

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TRIBHUVAN UNIVERSITY
INSTITUTE OF ENGINEERING
Examination Control Division
2078 Bhadra

Exam.	Regular		
Level	BE	Full Marks	80
Programme	All (Except BAR)	Pass Marks	32
Year / Part	II / I	Time	3 hrs.

Subject: - Engineering Mathematics III *(SH 501)*

- ✓ Candidates are required to give their answers in their own words as far as practicable.
- ✓ *Attempt All questions.*
- ✓ *The figures in the margin indicate **Full Marks**.*
- ✓ *Assume suitable data if necessary.*

1. Use the Properties of determinant to show that:

$$\begin{vmatrix} (a+b)^2 & ca & bc \\ ca & (b+c)^2 & ab \\ bc & ab & (c+a)^2 \end{vmatrix} = 2abc\,(a+b+c)^3$$ [5]

2. Define Hermitian and Skew-Hermitian of a square complex matrix. If A is any square matrix, prove that A + A* is Hermitian and A – A* is Skew – Hermitian matrix. [5]

3. Test the consistency of the system by matrix rank method and solve it completely if consistent: [5]

$x + 2y - z = 0,\ 2x + 3y + z = 10,\ 3x - y - 7z = 1$

4. Find the eigenvalues of the matrix $A = \begin{bmatrix} 1 & 0 & -1 \\ 1 & 2 & 1 \\ 2 & 2 & 3 \end{bmatrix}$ and use them to compute

(i) eigenvalues of A-1
(ii) determinant of A
(iii)eigenvalues of adj A [2+1+1+1]

5. Evaluate $\int_C \vec{F}.d\vec{r}$ where $\vec{F} = \text{Sin}y\,\vec{i} + x(1+\cos y)\vec{j}$ and C is the circular path given by $x^2 + y^2 = a^2, z = 0$. [5]

6. Evaluate $\iint_S \vec{F}.\vec{n}\,ds$ where $\vec{F} = yz\,\vec{i} + zx\,\vec{j} + xy\,\vec{k}$ where S is the surface of the sphere $x^2 + y^2 + z^2 = 1$ in the first octant. [5]

7. Apply Green's Theorem in plane to compute the area of the curve $\left(\frac{x}{a}\right)^{2/3} + \left(\frac{y}{b}\right)^{2/3} = 1$. [5]

8. State Gauss divergence theorem in vector calculus. Apply it to evaluate $\iint_S \left[(x^3 - yz)\vec{i} - 2x^2y\vec{j} + 2\vec{k}\right].\vec{n}\,ds$ where S denote the surface of the cube bounded by the planes x = 0, x = a, y = 0, y = a, z = 0, z = a. [1+4]

9. State the condition for existence property of Laplace transform. Find the Laplace transform of: (a) $\frac{1}{\sqrt{t}}$ (b) $\frac{1-\cos 2t}{t}$ [1+2+2]

10. State the convolution theorem for inverse Laplace transform and use it to find the inverse Laplace transform of $\frac{s}{(s^2+1)(s^2+4)}$. [5]

11. Solve the initial value problem by applying Laplace transform: [5]

$$y'' - 10y' + 9y = 5t,\ y(0) = -1,\ y'(0) = 2.$$

12. Obtain the Fourier series of $f(x) = x + x^2$ in $-\pi \leq x \leq \pi$. [5]

13. Express $f(x) = x^2$ as a half-range sine series in $0 < x < 3$. [5]

14. Solve following LPP by the Simplex method: [7]

Maximize, $P = x_1 + x_2$

Subject to : $2x_1 + x_2 \leq 16$

$x_1 \leq 6$

$x_2 \leq 10$

$x_1 \geq 0,\ x_2 \geq 0$

15. Solve following LPP by the Dual Method: [8]

Minimize, $C = 21x_1 + 50x_2$

Subject to : $2x_1 + 5x_2 \geq 12$

$3x_1 + 7x_2 \geq 17$

$x_1 \geq 0,\ x_2 \geq 0$

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TRIBHUVAN UNIVERSITY INSTITUTE OF ENGINEERING **Examination Control Division** **2078 Kartik**	**Exam.**	**Back**		
	Level	BE	**Full Marks**	80
	Programme	All (Except BAR)	**Pass Marks**	32
	Year / Part	II / I	**Time**	3 hrs.

Subject: - Engineering Mathematics III *(SH 501)*

- ✓ Candidates are required to give their answers in their own words as far as practicable.
- ✓ *Attempt **All** questions.*
- ✓ *The figures in the margin indicate **Full Marks**.*
- ✓ *Assume suitable data if necessary.*

1. If $\begin{vmatrix} a & a^2 & a^3-1 \\ b & b^2 & b^3-1 \\ c & c^2 & c^3-1 \end{vmatrix} = 0$; where $a \neq b \neq c$, apply the properties of determinants to show $abc = 1$. [5]

2. Define an orthogonal matrix. Prove that the product of two orthogonal matrices of the same order is also orthogonal. [5]

3. For the matrix = $\begin{bmatrix} 5 & 4 \\ 1 & 2 \end{bmatrix}$, find the modal matrix and the corresponding diagonal matrix. [5]

4. State Cayley-Hamilton theorem and verify the theorem for the square matrix $A = \begin{bmatrix} 1 & 3 & 7 \\ 4 & 2 & 3 \\ 1 & 2 & 1 \end{bmatrix}$. [5]

5. Prove that "for any simple closed curve C, the line integral $\int_A^B \vec{F}.d\vec{r}$ is independent of the path joining the points A and B in the region if and only if $\int_C \vec{F}.d\vec{r} = 0$. [5]

6. State Green's theorem in the plane. Using Green's theorem find the area of the hypocycloid $\left(\frac{x}{a}\right)^{2/3} + \left(\frac{y}{b}\right)^{2/3} = 1$. [5]

7. Evaluate $\iint_S \vec{F}.\vec{n}\,ds$ by Gauss' divergence theorem, where $\vec{F} = x\vec{i} - y\vec{j} + (z^2-1)\vec{k}$ and S is the cylinder formed by the surfaces $x^2 + y^2 = 4$, $z = 0$, $z = 1$. [5]

8. Verify Stoke's theorem for $\vec{F} = (x^2 - y^2)\vec{i} + 2xy\vec{j}$ taken over the rectangular bounded by the lines $x = 0$, $x = a$, $y = 0$, $y = b$. [5]

9. Define Laplace transform of f(t). Find the Laplace transform of:

   a) $t\,e^{-t}\cosh t$ b) $\frac{\sin t \sin 5t}{t}$ [1+1.5+2.5]

10. Find the inverse Laplace transform of:

    a) $\log\frac{S}{S+1}$ b) $\frac{1}{(S-2)(S^2+1)}$ [2.5+2.5]

11. Solve the initial value problem $y'' + 4y' + 3y = 0$, $y(0) = 3$, $y'(0) = 1$ by using Laplace transform. [5]

12. Find the Fourier series of $f(x) = 2x - x^2$ in (0, 2). [5]

13. Obtain the half range sine series for $f(x) = e^x$ in $0 < x < 1$. [5]

14. Use Simplex method to solve following LPP: [7]

    Maximize, $P = 50x_1 + 80x_2$
    Subject to : $x_1 + 2x_2 \leq 32$
    $3x_1 + 4x_2 \leq 84$
    $x_1, x_2 \geq 0$

15. Solve the following LPP by using big M method: [8]

    Maximize, $P = 2x + y$

    Subject to: $x + y \leq 10$
    $-x + y \geq 2$
    $x, y \geq 0$

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TRIBHUVAN UNIVERSITY
INSTITUTE OF ENGINEERING
Examination Control Division
2076 Chaitra

Exam.	Regular		
Level	BE	Full Marks	80
Programme	All (Except BAR)	Pass Marks	32
Year / Part	II / I	Time	3 hrs.

Subject: - Engineering Mathematics III *(SH 501)*

- ✓ Candidates are required to give their answers in their own words as far as practicable.
- ✓ *Attempt **All** questions.*
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1. Prove that $\begin{vmatrix} 1+a^2-b^2 & 2ab & -2b \\ 2ab & 1-a^2+b^2 & 2a \\ 2b & -2a & 1-a^2-b^2 \end{vmatrix} = (1+a^2+b^2)^3$ by using the properties of determinants. [5]
2. Prove that every square complex matrix can uniquely be expressed as a sum of a Hermitian and a skew-Hermitian matrix. [5]
3. Reduce the matrix $\begin{bmatrix} 1 & 0 & -5 & 6 \\ 3 & -2 & 1 & 2 \\ 5 & -2 & -9 & 14 \\ 4 & -2 & -4 & 8 \end{bmatrix}$ into normal form and hence find its rank. [5]
4. Find the eigen values and eigen vectors of the matrix $\begin{bmatrix} 2 & 0 & 1 \\ 0 & 2 & -1 \\ 0 & 0 & 2 \end{bmatrix}$ and also fnd its modal matrix. [5]
5. If $\vec{F} = 3x^2yz^2\,\vec{i} + x^3z^2\,\vec{j} + 2x^3yz\,\vec{k}$, show that $\int_c \vec{F}.d\vec{r}$ is independent of the path of integration. Hence evaluate the integral on any path C from (0, 0, 0) to (1, 2, 3). [5]
6. Verify Green's Theorem in plane for $\int_c [(x-y)\,dx + (x+y)\,dy]$ where c is the boundary of the region enclosed by $y^2 = x$ and $x^2 = y$. [5]
7. Evaluate $\iint_S \vec{F}.\vec{n}\,ds$ where $\vec{F} = 4x\,\vec{i} - 2y^2\,\vec{j} + z^2\,\vec{k}$ taken over the region bounded by the cylinder $x^2 + y^2 = 4$ and the planes $z = 0, z = 3$. [5]
8. Evaluate $\int_c \vec{F}.d\vec{r}$, where c is the rectangle bounded by the lines $x = \pm a, y = 0, y = n$ and $\vec{F} = (x^2 + y^2)\,\vec{i} - 2xy\,\vec{j}$. [5]
9. State the condition for existence of Laplace transform. Obtain the Laplace transform of:

   a) $\cos^3 2t$ (b) $\frac{\cos at - \cos bt}{t}$ [1+1.5+2.5]

10. Find the inverse Laplace transform of:

a) $\frac{s+3}{(s^2+6s+13)^2}$ b) $\frac{e^{-2s}}{(s+1)(s^2+2s+2)}$ [2+3]

11. Solve the differential equation $y''+2y'-3y = \sin t$ under the conditions $y(0) = y'(0) = 0$ by using Laplace transform. [5]

12. Obtain the Fourier series to represent the function $f(x) = e^x$ for $-\pi \le x \le \pi$. [5]

13. Obtain the half range cosine series for the function $f(x) = x\sin x$ in the interval $(0, \pi)$. [5]

14. Use Simplex method to solve following LPP:

Maximize, $P = 30x_1 + x_2$

Subject to : $2x_1 + x_2 \le 10$

$x_1 + 3x_2 \le 10$

$x_1, x_2 \ge 0$ [7]

15. Use Big M method to solve following LPP:

16. Minimize, $Z = 4x_1 + 2x_2$

Subject to : $3x_1 + x_2 \ge 27$

$-x_1 - x_2 \le -21$

$x_1 + 2x_2 \ge 30$

$x_1, x_2 \ge 0$ [8]

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TRIBHUVAN UNIVERSITY
INSTITUTE OF ENGINEERING
Examination Control Division
2076 Ashwin

Exam.	Back		
Level	BE	Full Marks	80
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Year / Part	II / I	Time	3 hrs.

Subject: - Engineering Mathematics III *(SH 501)*

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- ✓ *The figures in the margin indicate __Full Marks__.*
- ✓ *Assume suitable data if necessary.*

1. Prove that: $\begin{vmatrix} (b+c)^2 & c^2 & b^2 \\ c^2 & (c+a)^2 & a^2 \\ b^2 & a^2 & (a+b)^2 \end{vmatrix} = 2(ab+bc+ca)^2$ [5]

2. Prove that the necessary and sufficient condition for a square matrix A to possess an inverse is that $|A| \neq 0$. [5]

3. Find the rank of the matrix $\begin{bmatrix} 2 & -2 & 0 & 6 \\ 4 & 2 & 0 & 2 \\ 1 & -1 & 0 & 3 \\ 1 & -2 & 1 & 2 \end{bmatrix}$ by reducing it to normal form. [5]

4. State any two properties of eigen values of a matrix. Obtain eigen values and eigen vectors of the matrix $\begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$ [1+4]

5. Prove that the line integral $\int_A^B \vec{F}.d\vec{r}$ is independent of path joining any two points A and B in the region if and only if $\oint_C \vec{F}.d\vec{r} = 0$ for any simple closed curve C in the region. [5]

6. State Green's Theorem and use it to find the area of the curve $\left(\frac{x}{a}\right)^{2/3} + \left(\frac{y}{b}\right)^{2/3} = 1$. [1+4]

7. Use Gauss' divergence theorem to evaluate $\iint_S \vec{F}.\vec{n}ds$ where $\vec{F} = (2xy+z)\vec{i} + y^2\vec{j} - (x+3y)\vec{k}$ and S is the surface bounded by the plane 2x+3y+z=6, x=0, y=0, z=0. [5]

8. Verify Stoke's Theorem for the vector field $\vec{F} = (2x-y)\vec{i} - yz^2\vec{j} - y^2z\vec{k}$ over the upper half of the sphere $x^2+y^2+z^2=1$ bounded by its projection on xy-plane. [5]

9. Find the Laplace transform of: [2+3]

   i) $t^2\cos at$

   ii) $\frac{1-\cosh(at)}{t}$

10. Find the inverse Laplace transform of : [2+3]

i) $\frac{e^{-\pi s}(s+1)}{s^2+2s+2}$

ii) $\tan^{-1}\frac{2}{s}$

11. Solve the differential equation $y''+3y'+2y=e^{-t}$, $y(0)=y'(0)=0$ by applying Laplace transform. [5]

12. Find the Fourier Series of the function $f(x)=|\sin x|$ for $-\pi \le x \le \pi$. [5]

13. If $f(x) = lx-x^2$ in $(0,1)$, show that the half range sine series for $f(x)$ is $\frac{8l^2}{\pi^3}\sum_{n=0}^{\infty}\frac{1}{(2n+1)^3}\sin(2n+1)\frac{\pi x}{l}$. [5]

14. Find the maximum and minimum values of the function $z=20x+10y$ subject to: $x+2y\le40$, $3x+y\ge30$, $4x+3y \ge 60$, $x,y\ge0$ by graphical method. [5]

15. Solve the following linear programming problem using big M method:

Maximize $P=2x_1+5x_2$

subject to : $x_1+2x_2 \le 18$

$2x_1+x_2 \ge 21$

$x_1, x_2 \ge 0$. [10]

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TRIBHUVAN UNIVERSITY
INSTITUTE OF ENGINEERING
Examination Control Division
2075 Chaitra

Exam.	Regular / Back		
Level	BE	Full Marks	80
Programme	All except BAR	Pass Marks	32
Year / Part	II / I	Time	3 hrs.

Subject: - Engineering Math III *(SH 501)*

- ✓ Candidates are required to give their answers in their own words as far as practicable.
- ✓ *Attempt **All** questions.*
- ✓ *The figures in the margin indicate **Full Marks**.*
- ✓ *Assume suitable data if necessary.*

1. If $\begin{vmatrix} a & a^2 & a^3-1 \\ b & b^2 & b^3-1 \\ c & c^2 & c^3-1 \end{vmatrix} = 0$, where $a \neq b \neq c$ show that $abc=1$. [5]
2. If A is a square matrix of order n, prove that $A(adj.\ A) = (adj.\ A)A = |A|I_n$, where I_n is a unit matrix having same order as A. [5]
3. Test the consistency of the system by matrix rank method and solve completely if found consistent: $x+2y-z=3$, $2x+3y+z=10$, $3x-y-7z=1$ [5]
4. State Cayley-Hemilton Thorem and verify it for the matrix $A=\begin{bmatrix} 1 & 0 & -1 \\ 1 & 2 & 1 \\ 2 & 2 & 3 \end{bmatrix}$ [1+4]
5. A vector field is given by $\vec{F} = \sin y\vec{i} + x(1+\cos y)\vec{j}$. Evaluate the line integral $\int_c \vec{F}.d\vec{r}$ over the circular path c given by $x^2+y^2=a^2$, $z=0$. [5]
6. State and prove Green's Theorem in plane. [1+4]
7. Evaluate $\iint_s \vec{F}.\vec{n}\,ds$ for $\vec{F} = yz\vec{i} + zx\vec{j} + xy\vec{k}$ where S is the surface of the sphere $x^2+y^2+z^2=1$ in the first octant. [5]
8. State Stoke's theorem. Evaluate $\oint_c (xydx + xy^2dy)$ by Stoke's theorem taking c to be a square in the xy-plane with vertices (1,0),(-1,0),(0,1) and (0,-1). [1+4]
9. Find the Laplace transform of : [2+3]
   i) $te^{-t}\sin t$
   ii) $\dfrac{\cos 2t - \cos 3t}{t}$
10. Find the inverse Laplace transform of : [2+3]
    i) $\dfrac{s+2}{(s+1)^4}$
    ii) $\cot^{-1}(s+1)$
11. Solve the differential equation $y''+y=\sin 3t$, $y(0)=y'(0)=0$ by using Laplace transform. [5]
12. Define Fourier Series for a function f(x). Obtain Fourier series for $f(x)=x^3$; $-\pi \leq x \leq \pi$. [5]
13. Express $f(x)=e^x$ as the half range Fourier Sine series in $0<x<1$. [5]
14. Find the maximum and minimum values of the function $z = 50x_1 + 80x_2$ subject to: $x_1 + 2x_2 \leq 32$, $3x_1 + 4x_2 \leq 84$, $x_1x_2 \geq 0$; by graphical method. [5]
15. Solve the following Linear Programming problem using big M method: [10]
    Maximize $P= 2x_1+x_2$
    Subject to : $x_1+x_2 \leq 10$
    $-x_1+x_2 \geq 2$
    $x_1,x_2 \geq 0$

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02 TRIBHUVAN UNIVERSITY
INSTITUTE OF ENGINEERING
Examination Control Division
2075 Ashwin

Exam.	Back		
Level	BE	Full Marks	80
Programme	All (Except B.Arch.)	Pass Marks	32
Year / Part	II / I	Time	3 hrs.

Subject: - Engineering Mathematics III *(SH501)*

- ✓ Candidates are required to give their answers in their own words as far as practicable.
- ✓ *Attempt **All** questions.*
- ✓ *The figures in the margin indicate **Full Marks**.*
- ✓ *Assume suitable data if necessary.*

1. Define the determinant as a function and using its properties. Show that

$$\begin{vmatrix} b+c & c+a & a+b \\ q+r & r+p & p+q \\ y+z & z+x & x+y \end{vmatrix} = 2\begin{vmatrix} a & p & x \\ b & q & y \\ c & r & z \end{vmatrix}$$ [5]

2. If A and B are orthogonal matrices of same order, prove that the product AB is also orthogonal. [5]

3. Test the consistency of the system $x-2y+2z=4$, $3x+y+4z=6$ and $x+y+z=1$ and solve completely if found consistent. [5]

4. For a matrix $A=\begin{pmatrix} 5 & 4 \\ 1 & 2 \end{pmatrix}$, find the modal matrix and the corresponding diagonal matrix. [5]

5. Prove that line integral $\int_A^B \vec{F}.d\vec{r}$ is independent of path joining any two points A and B in the region if and only if $\int_C \vec{F}.d\vec{r}=0$ for any simple closed curve C in the region. [5]

6. Verify Green's theorem in the plane for $\int_C [(3x^2-8y^2)dx+(4y-6xy)dy]$ where C is region bounded by $y=x^2$ and $x=y^2$. [5]

7. Evaluate $\iint_S \vec{F}.\vec{n}\,ds$ where $\vec{F}=6z\vec{i}-4\vec{j}+y\vec{k}$ and S is the region of the plane $2x+3y+6z=12$ bounded in the first octant. [5]

8. Evaluate using Gauss divergence theorem, $\iint_S \vec{F}.\vec{n}\,ds$ where $\vec{F}=x^2y\vec{i}+xy^2\vec{j}+2xyz\vec{k}$ and S is the surface bounded by the planes $x=0$, $y=0$, $z=0$, $x+2y+z=2$. [5]

9. Obtain the Fourier Series to represent $f(x)=x-x^2$ from $x=-\pi$ to $x=\pi$ and deduce that

$$\frac{\pi^2}{12}=\frac{1}{1^2}-\frac{1}{2^2}+\frac{1}{3^2}-\frac{1}{4^2}+\dots\dots\dots$$ [5]

10. Obtain the half range Fourier Sine Series for $f(x)=\pi-x$ in the range $0<x<\pi$. [5]

11. State the conditions for existence of Laplace transform. Obtain the Laplace transform of:

(i) $e^{2t}\cos^3 2t$ (ii) $\frac{\cos 2t-\cos 3t}{t}$ [1+2+2]

12. Find the inverse Laplace transform of:

(i) $\frac{1}{(S-2)(S^2+1)}$ (ii) $\cot^{-1}(S+1)$ [2.5+2.5]

13. Solve the following intial value problem by using Laplace transform:

$y'' + 4y' + 3y = e^t$, $y(0) = 0; y'(0) = 2$ [5]

14. Graphically maximize $Z = 7x_1 + 10x_2$

Subject to constraints:

$$3x_1 + x_2 \le 9$$
$$x_1 + 2x_2 \le 8$$
$$x_1, x_2 \ge 0.$$

[5]

15. Solve the following linear Programming Problem by simple method:

Maximize: $Z = 3x_1 + 5x_2$

Subject to:

$$3x_1 + 2x_2 \le 18$$
$$x_1 \le 4, \ x_2 \le 6$$
$$x_1, x_2 \ge 0.$$

[10]

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01 TRIBHUVAN UNIVERSITY

INSTITUTE OF ENGINEERING

Examination Control Division

2074 Chaitra

Exam.	Regular		
Level	BE	Full Marks	80
Programme	All (Except B.Arch.)	Pass Marks	32
Year / Part	II / I	Time	3 hrs.

Subject: - Engineering Mathematics III *(SH501)*

- ✓ Candidates are required to give their answers in their own words as far as practicable.
- ✓ *Attempt **All** questions.*
- ✓ *The figures in the margin indicate **Full Marks**.*
- ✓ *Assume suitable data if necessary.*

1. If $\begin{vmatrix} a & a^2 & a^3-1 \\ b & b^2 & b^3-1 \\ c & c^2 & c^3-1 \end{vmatrix} = 0$ where $a \neq b \neq c$; apply properties of determinant to show abc = 1. [5]

2. If A be an $n \times n$ matrix, prove that

   $\text{Adj}(A) . A = A . (\text{Adj}A) = |A| I$ where I is an $n \times n$ unit matrix. [5]

3. Find the rank of the following matrix by reducing it into normal form:

   $$\begin{pmatrix} 3 & 1 & 4 \\ 0 & 5 & 8 \\ -3 & 4 & 4 \\ 1 & 2 & 4 \end{pmatrix}$$ [5]

4. Find the modal matrix for the matrix

   $$A = \begin{pmatrix} 2 & 1 & 1 \\ -2 & 1 & 3 \\ 2 & 1 & -1 \end{pmatrix}$$ [5]

5. State and prove Green's theorem in plane. [5]

6. Find the total work done in moving the particle in a force field given by $\vec{F} = \text{Sin}y\,\vec{i} + x(1+\cos y)\,\vec{j}$ over the circular path $x^2 + y^2 = a^2,\ z = 0$. [5]

7. Evaluate $\iint_s \vec{F}.d\vec{s}$ where $\vec{F} = x\vec{i} - y\vec{j} + z\vec{k}$ and s is the surface of the cylinder $x^2 + y^2 = a^2, 0 < z < b$. [5]

8. Verify Stoke's theorem for $\vec{F} = (x^2 + y^2)\vec{i} - 2xy\vec{j}$ taken round the rectangle bounded by the lines $x = \pm a, y = 0, y = b$. [5]

9. Obtain Fourier series for $f(x) = x^3$ in the interval $-\pi \leq x \leq \pi$. [5]

10. Express $f(x) = e^x$ as a half range Fourier Cosine Series in $0 < x < 1$. [5]

11. State existence theorem for Laplace Transform. Obtain the Laplace transform of

    a) $te^{-t}\sin t$ b) $\dfrac{e^{-at} - e^{-bt}}{t}$ 1+2+2]

12. Find the inverse Laplace transform of:

a) $\frac{1}{s^2 - 5s + 6}$ b) $\tan^{-1}\frac{2}{s}$ [2+5.+2.5]

13. By using Laplace transform, solve the initial value problem:

$y'' + 2y = r(t),\ y(0) = y'(0) = 0$
Where $r(t) = 1,\ 0 < t < 1$
$= 0$, otherwise

[5]

14. Graphically maximize $Z = 5x_1 + 3x_2$ Subject to constraints

$x_1 + 2x_2 \leq 50$
$2x_1 + x_2 \leq 40.$
$x_1, x_2 \geq 0$

[5]

15. Solve the following Linear Programming Problem by simple method:

Maximize : $Z = 4x + 3y$

Subject to : $2x + 3y \leq 6$
$-x + 2y \leq 3$
$2y \leq 5$
$2x + y \leq 4$
$x, y \geq 0.$

[10]

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01 TRIBHUVAN UNIVERSITY

INSTITUTE OF ENGINEERING

Examination Control Division

2074 Ashwin

Exam.	Back		
Level	BE	Full Marks	80
Programme	ALL (Except B. Arch)	Pass Marks	32
Year / Part	II / I	Time	3 hrs.

Subject: - Engineering Mathematics III *(SH501)*

- ✓ Candidates are required to give their answers in their own words as far as practicable.
- ✓ *Attempt **All** questions.*
- ✓ *The figures in the margin indicate **Full Marks**.*
- ✓ *Assume suitable data if necessary.*

1. Use properties of determinant to show [5]

$$\begin{vmatrix} x^2 & x^2-(y-z)^2 & yz \\ y^2 & y^2-(z-x)^2 & zx \\ z^2 & z^2-(x-y)^2 & xy \end{vmatrix} = (x-y)(y-z)(z-x)(x+y+z)(x^2+y^2+z^2)$$

2. Prove that every square matrix can be uniquely expressed as the sum of symmetric and a skew symmetric matrix. [5]

3. Define eigen values and eigen vectors in terms of linear transformation with matrices as operator. Find eigen values of the matrix. [2+3]

$$\begin{pmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{pmatrix}$$

4. Test the consistency of the system $x+y+z=3,\ x+2y+3z=4,\ 2x+3y+4z=7$ by using rank of matrix method and solve if consistent. [5]

5. If $\vec{F}$ is the gradient of some scalar point functions ϕ i.e $\vec{F}=\nabla\phi$, prove that the line integral is independent of the path joining any two points in the region and conversely. [5]

6. Evaluate $\iint_S \vec{F}.\vec{n}\,ds$. where $\vec{F}=xy\,\vec{i}-x^2\,\vec{j}+(x+z)\vec{K}$ and S is the region of the plane $2x+2y+z=6$ bounded in the first quadrant. [5]

7. State and prove Green's theorem in plane. [5]

8. Apply Gauss' divergence theorem to evaluate $\iint_S \left[(x^3-yz)\vec{i}-2x^2y\,\vec{j}+2\vec{K}\right].\vec{n}\,ds$, where S is the surface of the cube bounded by the planes x = 0, x = a, y = 0, y = a, z = 0, z = a. [5]

9. Expand f(x) = x sinx as a Fourier series in $-\pi \le x \le \pi$. [5]

10. Obtain half range cosine series for f(x) = x in the interval $0 \le x \le \pi$. [5]

11. Find the Laplace transform of: [3+2]

    i) $t^2\cos at$

    ii) $\dfrac{\sin t}{t}$

12. State convolution theorem for inverse Laplace transform and use it to find the inverse Laplace transform of $\frac{S}{(S^2+4)(S^2+9)}$ [1+4]

13. Solve the following initial value problem by using Laplace transform: [5]

$y''+2y'-3y=\sin t,\ y(0)=y'(0)=0$

14. Graphically maximize [5]

$Z=7x_1+10x_2$

Subject to constraints,

$3x_1+x_2\le 9$

$x_1+2x_2\le 8$

$x_1,x_2\ge 0$

15. Solve the following LPP by simplex method using duality of: [10]

Minimize Z = 20x+50y

Subject to:

$2x+5y\ge 12$

$3x+7y\ge 17$

$x,y\ge 0$

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01 TRIBHUVAN UNIVERSITY

INSTITUTE OF ENGINEERING

Examination Control Division

2073 Shrawan

Exam.	**New Back (2066 & Later Batch)**		
Level	BE	Full Marks	80
Programme	ALL (Except B. Arch)	Pass Marks	32
Year / Part	II / I	Time	3 hrs.

Subject: - Engineering Mathematics II *(SH501)*

- ✓ Candidates are required to give their answers in their own words as far as practicable.
- ✓ *Attempt **All** questions.*
- ✓ *The figures in the margin indicate **Full Marks**.*
- ✓ *Assume suitable data if necessary.*

1. Use properties of determinants to prove: $\begin{vmatrix} a^2 & bc & ac+c^2 \\ a^2+ab & b^2 & ac \\ ab & b^2+bc & c^2 \end{vmatrix} = 4a^2b^2c^2$ [5]

2. Prove that the necessary and sufficient condition for a square matrix A to posses an inverse is that the matrix A should be non singular. [5]

3. Find the rank of the matrix $\begin{pmatrix} 1 & 3 & -2 & 1 \\ 1 & 1 & 1 & 1 \\ 2 & 0 & -3 & 2 \\ 3 & 3 & -3 & 3 \end{pmatrix}$ [5]

   by reducing it into normal form.

4. Find the eigenvalues and eigenvectors of the matrix $\begin{pmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 0 & 0 & 1 \end{pmatrix}$ [4+1]

   Give an example showing importance of eigenvectors.

5. Show that $\vec{F} = (2x+z^2)\vec{i} + Z\vec{j} + (y+2xz)\vec{K}$ is irrotational and find its scalar potential. [5]

6. State and prove Green's Theorem in plane. [5]

7. Evaluate $\iint_S \vec{F}.\vec{n}\, ds$, where $\vec{F} = yz\vec{i} + zx\vec{j} + xy\vec{k}$ and S is the surface of the sphere $x^2+y^2+z^2=1$ in the first octant. [5]

8. Evaluate $\int_c xydx + xy^2dy$ by applying stokes theorem where C is the square in xy-plane with vertices (1,0), (-1,0), (0,1), (0,-1) [5]

9. Find the Laplace transform of : [2+3]

   i) $te^{2t}\sin 3t$

   ii) $\dfrac{e^{-t}\sin t}{t}$

10. Find the inverse Laplace transform of : [2+3]

i) $\frac{s+2}{s^2-4s+13}$

ii) $\log\left(\frac{s+a}{s-a}\right)$

11. Solve the following initial value problem using Laplace transform: [5]

$x''+4x'+4x=6e^{-t}, \quad x(0)=-2, \quad x'(0)=-8$

12. Find the Fourier series representation of $f(x)=|x|$ in $[-\pi, \pi]$ [5]

13. Obtain the half range Fourier Sine Series for the function $f(x) = x^2$ in the interval (0, 3). [5]

14. Apply Graphical method to maximize, [5]

$Z=5x_1+3x_2$

Subject to the constraints:

$x_1+2x_2 \le 50$

$2x_1+x_2 \le 40$

$x_1 \ge 0,\ x_2 \ge 0$

15. Solve the following Linear Programming Problem by Simplex method: [10]

Maximize: $Z=15x_1+10x_2$

Subject to: $x_1+3x_2 \le 10$

$2x_1+x_2 \le 10$

$x_1 \ge 0,\ x_2 \ge 0$

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01 TRIBHUVAN UNIVERSITY

INSTITUTE OF ENGINEERING

Examination Control Division

2072 Chaitra

Exam.	Regular		
Level	BE	Full Marks	80
Programme	All (Except B. Arch)	Pass Marks	32
Year / Part	II / I	Time	3 hrs.

Subject: - Engineering Mathematics III *(SH501)*

- ✓ Candidates are required to give their answers in their own words as far as practicable.
- ✓ *Attempt **All** questions.*
- ✓ *The figures in the margin indicate **Full Marks**.*
- ✓ *Assume suitable data if necessary.*

1. Use properties of determinants to prove: [5]

$$\begin{vmatrix} a^2+1 & ba & ca & da \\ ab & b^2+1 & cb & db \\ ac & bc & c^2+1 & dc \\ ad & bd & cd & d^2+1 \end{vmatrix} = 1+a^2+b^2+c^2+d^2$$

2. Show that every square matrix can be uniquely expressed as the sum of symmetric and Skew-Symmetric matrices. [5]

3. Test the consistency of the system $x+y+z=3, x+2y+3z=4$ and $2x+3y+4z=7$ and solve completely if found consistent. [5]

4. State Cayley-Hamilton theorem and verify it for the matrix; $A=\begin{pmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{pmatrix}$ [1+4]

5. Prove that " The line integral $\int_c \vec{F}.d\vec{r}$ of a continuous function $\vec{F}$ defined in a region R is independent of path C joining any two points in R if and only if there exists a single valued scalar function ϕ having first order partial derivatives such that $\vec{F}=\nabla\phi$". [5]

6. State Green's theorem and use it to find the area of astroid $x^{2/3}+y^{2/3}=a^{2/3}$. [5]

7. Evaluate $\iint_s \vec{F}.\vec{n}\, ds$, where $\vec{F}=x^2\vec{i}+y^2\vec{j}+z^2\vec{k}$ and 's' is the surface of the plane $x+y+z=1$ between the co-ordinate planes. [5]

8. Apply Gauss' divergence theorem to evaluate $\iint_s \vec{F}.\vec{n}\, ds$ where

   $\vec{F}=(x^3-yz)\vec{i}-2x^2y\vec{j}+2\vec{k}$ and 's' is the surface the cube bounded by the planes $x=0, x=a, y=0, y=a, z=0, z=a$. [5]

9. Find the Laplace transform of: [2+3]

i) $tSin^2 3t$

ii) $\frac{Sin2t}{t}$

10. Find the inverse Laplace transform of: [2+3]

i) $\frac{1}{s^2 - 3s + 2}$

ii) $\frac{1}{s(s+1)^3}$

11. Apply Laplace transform to solve the differential equation: [5]

$y''+2y'+5y = e^{-t}\sin t, \quad x(0) = 0, x'(0) = 1$

12. Find a Fourier series to represent $f(x) = x - x^2$ from $x = -\pi$ to $x = \pi$. Hence show that

$\frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + ... = \frac{\pi^2}{12}$ [5]

13. Develop $f(x) = \sin\left(\frac{\pi x}{l}\right)$ in half range Cosine Series in the range $0 < x < l$. [5]

14. Graphically maximize, [5]

$Z = 7x_1 + 10x_2$

Subject to constraints,

$3x_1 + x_2 \leq 9$

$x_1 + 2x_2 \leq 8$

$x_1 \geq 0, x_2 \geq 0$

15. Solve the following LPP using simplex method. [10]

Maximize: $P = 50x_1 + 80x_2$

Subject to: $x_1 + 2x_2 \leq 32$

$3x_1 + 4x_2 \leq 84$

$x_1 \geq 0, x_2 \geq 0$

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01 TRIBHUVAN UNIVERSITY	Exam.	Regular		
INSTITUTE OF ENGINEERING	Level	BE	Full Marks	80
Examination Control Division	Programme	All (Except B.Arch.)	Pass Marks	32
2071 Chaitra	Year / Part	II / I	Time	3 hrs.

Subject: - Engineering Mathematics III *(SH501)*

- ✓ Candidates are required to give their answers in their own words as far as practicable.
- ✓ *Attempt __All__ questions.*
- ✓ *The figures in the margin indicate __Full Marks__.*
- ✓ *Assume suitable data if necessary.*

1. Using the properties, evaluate the determinant: [5]

$$\begin{vmatrix} 1 & a & a^2 & a^3+bcd \\ 1 & b & b^2 & b^3+cda \\ 1 & c & c^2 & c^3+abd \\ 1 & d & d^2 & d^3+abc \end{vmatrix}$$

2. Prove that every square matrix can uniquely be expressed as the sum of a symmetric and a skew symmetric matrix. [5]

3. Test the consistency of the system: [5]

   $x-6y-z=10,\ 2x-2y+3z=10,\ 3x-8y+2z=20$

   And solve completely, if found consistent.

4. Find the eigen values and eigenvecters of the matrix $\begin{pmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{pmatrix}$. [5]

5. Using the line integral, compute the workdone by the force [5]

   $\vec{F}=(2x-y+2z)\vec{i}+(x+y-z)\vec{j}+(3x-2y-5z)\vec{k}$

   when it moves once around a circle $x^2+y^2=4; z=0$

6. State and prove Green's Theorem in plane. [5]

7. Verify Stoke's theorem for $\vec{F}=(x^2+y^2)\vec{i}-2xy\vec{j}$ taken around the rectangle bounded by the lines $x=\pm a$, $y=0$, $y=b$. [5]

8. Evaluate $\iint_S \vec{F}.\vec{n}\, ds$ where $\vec{F}=(2xy+z)\vec{i}+y^2\vec{j}-(x+3y)\vec{K}$ by Gauss divergence theorem; where S is surface of the plane $2x+2y+z=6$ in the first octant bounding the volume V. [5]

9. Find the Laplace transform of the following: [2.5×2]

   a) $te^{-2t}\cos t$

   b) $\text{Sinhat}.\cos t$

10. Find the inverse Laplace transform of : [2.5×2]

a) $\frac{1}{S(S+1)}$

b) $\frac{S^2}{(S^2+b^2)^2}$

11. Solve the differential equation $y''+2y'+5y = e^{-t}\sin t, y(0)=0, y'(0)=1$, by using Laplace transform. [5]

12. Expand the function $f(x) = x \sin x$ as a Fourier series in the interval $-\pi \leq x \leq \pi$. [5]

13. Obtain half range sine series for the function $f(x) = x - x^2$ for $0 < x < 1$. [5]

14. Graphically maximize and minimize [5]

$z = 9x + 40y$ subjected to the constraints

$y - x \geq 1, y - x \leq 3, 2 \leq x \leq 5$

15. Solve the following Linear Programming Problem by Simplex method: [10]

Maximize, $P = 20x_2 - 5x_1$

Subjected to, $10x_2 - 2x_1 \leq 5$

$2x_1 + 5x_2 \leq 10$ and $x_1, x_2 \geq 0$

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01 TRIBHUVAN UNIVERSITY
INSTITUTE OF ENGINEERING
Examination Control Division
2070 Chaitra

Exam.	Regular		
Level	BE	Full Marks	80
Programme	All (Except B.Arch)	Pass Marks	32
Year / Part	II / I	Time	3 hrs.

Subject: - Mathematics III *(SH501)*

- ✓ Candidates are required to give their answers in their own words as far as practicable.
- ✓ *Attempt **All** questions.*
- ✓ *The figures in the margin indicate **Full Marks**.*
- ✓ *Assume suitable data if necessary.*

1. Using the properties of determinant prove [5]

$$\begin{vmatrix} (b+c)^2 & a^2 & a^2 \\ b^2 & (c+a)^2 & b^2 \\ c^2 & c^2 & (a+b)^2 \end{vmatrix} = 2abc(a+b+c)^3$$

2. Prove that $(AB)^T = B^T A^T$ where A is the matrix of size m×p and B is the matrix of size p×n [5]

3. Find the rank of the following matrix by reducing normal form. $\begin{bmatrix} 1 & 3 & -2 & 1 \\ 1 & 1 & 1 & 1 \\ 2 & 0 & -3 & 2 \\ 3 & 3 & -3 & 3 \end{bmatrix}$ [5]

4. Find the eigen values and eigen vectors of the following matrix. $\begin{bmatrix} 2 & 0 & 1 \\ 0 & 2 & -1 \\ 0 & 0 & 2 \end{bmatrix}$ [5]

5. Prove that the line intergral $\int_A^B \vec{F}.d\vec{r}$ is independent of the path joining any two points A and B in a region if $\int_c \vec{F}.d\vec{r} = 0$ for any simple closed curve C in the region. [5]

6. Evaluate $\iint_s \vec{F}.\hat{n}\, ds$ where $\vec{F} = x^2\vec{i} + y^2\vec{j} + z^2\vec{k}$ and S is the finite plane x + y + z = 1 between the coordinate planes. [5]

OR

Evaluate $\iint_s \vec{F}.\hat{n}\, ds$ for $\vec{F} = yz\vec{i} + zx\vec{j} + xy\vec{k}$ where S is the surface of sphere $x^2 + y^2 + z^2 = 1$ in the first octant.

7. Evaluate, $\iint_s \vec{F}.\hat{n}\, ds$ for $\vec{F} = x\vec{i} - y\vec{j} + (z^2 - 1)\vec{k}$ where S is the surface bounded by the cylinder $x^2 + y^2 = 4$ and the planes z = 0 and z = 1 [5]

8. Verify the stoke's theorem for $\vec{F} = (2x - y)\vec{i} - yz^2\vec{j} - y^2z\vec{k}$ where S is the upper part of the sphere $x^2 + y^2 + z^2 = a^2$ C is its boundary. [5]

9. Find the Laplace transform of (a) $t^2 \sin zt$ and (b) $\frac{1-e^t}{t}$ [2.5×2]

10. Find the inverse Laplace transform of (a) $\frac{2s+3}{s^2+5s-6}$ (b) $\frac{s^3}{s^4-a^4}$ [2.5×2]

11. Solve the following differential equation by using Laplace transform [5]

$y'' + y' - 2y = x$, $y(0) = 1$, $y'(0) = 0$

12. Obtain the Fourior series for $f(x) = x^2$ in the interval $-\pi < x < \pi$ and hence prove that

$$\sum \frac{1}{x^2} = \frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} \ldots\ldots = \frac{\pi^2}{6}$$ [5]

13. Obtain half range sine series for $f(x) = \pi x - x^2$ in $(0, \pi)$ [5]

14. Graphically minimize $z = 4x_1 + 3x_2 + x_3$ [5]

Subject to $x_1 + 2x_2 + 4x_3 \geq 12$

$3x_1 + 2x_2 + x_3 \geq 8$ and $x_1, x_2, x_3 \geq 0$

15. Minimize $z = 8x_1 + 9x_2$ [10]

Subject to $x_1 + 3x_2 \geq 4$

$2x_1 + x_2 \geq 5$ with $x_1, x_2 \geq 0$

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62 TRIBHUVAN UNIVERSITY
INSTITUTE OF ENGINEERING
Examination Control Division.
2069 Ashad

Exam.	New Back (2066 & Later Batch)		
Level	BE	Full Marks	80
Programme	All (Except B. Arch.)	Pass Marks	32
Year / Part	II / I	Time	3 hrs.

Subject: - Engineering Mathematics III *(SH 501)*

- ✓ Candidates are required to give their answers in their own words as far as practicable.
- ✓ *Attempt **All** questions.*
- ✓ *The figures in the margin indicate **Full Marks**.*
- ✓ *Assume suitable data if necessary.*

1. Find the value of the determinant: [5]

$$\begin{vmatrix} 1 & a & a^2 & a^3+bcd \\ 1 & b & b^2 & b^3+cda \\ 1 & c & c^2 & c^3+dab \\ 1 & d & d^2 & d^3+abc \end{vmatrix}$$

2. Prove that every square matrix can be uniquely expressed as the sum of a symmetric and a skew-symmetric matrices. [5]

3. Find the rank of matrix: $\begin{bmatrix} 1 & 3 & -2 & 1 \\ 1 & 1 & 1 & 1 \\ 2 & 0 & -3 & 2 \\ 3 & 3 & -3 & 3 \end{bmatrix}$ reducing to echelon form. [5]

4. Verify Cayley-flamiltan theorem for the matrix: $\begin{bmatrix} 1 & 3 & 7 \\ 4 & 2 & 3 \\ 1 & 2 & 1 \end{bmatrix}$. [5]

5. Find the Laplace transforms of: (a) $te^{-t}\sin t$ (b) $\dfrac{e^{at}-\cos 6t}{t}$ [5]

6. If $L[f(t)]=F(s)$, then prove that $L[f^1(t)]=SF(s)-f(o)$. [5]

7. Use Laplace transform to solve: $x''+2x'+5x=e^{-t}\sin t$ given $x(0)=0$; $x'(0)=1$. [5]

8. Obtain the Fourier series for $f(x)=x^3$ in the interval $-\pi\leq x\leq\pi$. [5]

9. Obtain half-range sine series for e^x in (0, 1). [5]

10. Maximize $z=2x_1+3x_2$ subject to constraints $x_1-x_2\leq 2$, $x_1+x_2\geq 4$ and $x_1, x_2\geq 0$ graphically. [5]

11. Solve the linear programming problems by simplex method constructing the duality [10]

    Minimize $Z = 3x_1+2x_2$
    Subject to $2x_1+4x_2\geq 10$
    $4x_1+2x_2\geq 10$
    $x_2\geq 4$ and $x_1, x_2\geq 0$

12. Prove that $\vec{F} = (2xz^3 + 6y)\vec{i} + (6x - 2yz)\vec{j} + (3x^2z^2 - y^2)\vec{k}$ is conservative vector field and find its scalar potential function. [5]

13. Evaluate $\iint_S \vec{F}.\hat{n}\, ds$ where $\vec{F} = x^2\vec{i} + y^2\vec{j} + z^2\vec{k}$ and S is the finite plane x+y+z=1 between the co-ordinate planes. [5]

14. Using Green's theorem, find the area of the hypocycloid $\frac{x^{2/3}}{a^{2/3}} + \frac{y^{2/3}}{b^{2/3}} = 1.$ [5]

15. Evaluate $\iint_S \vec{F}.\hat{n}\, ds$ where $\vec{F} = 2x\vec{i} + 3y\vec{j} + 4z\vec{k}$ and S is the surface of sphere $x^2+y^2+z^2=1$ by Gauss divergence theorem. [5]

OR

Verify Stoke's theorem for $\vec{F} = 2y\vec{i} + 3x\vec{j} - z^2\vec{k}$ where S is the upper half of the sphere $x^2+y^2+z^2=9$ and 'C' is its boundary. [5]

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02 TRIBHUVAN UNIVERSITY

INSTITUTE OF ENGINEERING

Examination Control Division

2069 Chaitra

Exam.	Regular		
Level	BE	Full Marks	80
Programme	All (Except B.Arch)	Pass Marks	32
Year / Part	II / I	Time	3 hrs.

Subject: - Engineering Mathematics III *(SH501)*

- ✓ Candidates are required to give their answers in their own words as far as practicable.
- ✓ *Attempt **All** questions.*
- ✓ *The figures in the margin indicate **Full Marks**.*
- ✓ *Assume suitable data if necessary.*

1. Find the value of the determinant $\begin{vmatrix} a^2 & a^2-(b-c)^2 & bc \\ b^2 & b^2-(c-a)^2 & ca \\ c^2 & c^2-(a-b)^2 & ab \end{vmatrix}$ [5]

2. Show that the matrix $B^\theta AB$ is Hermitian or skew-Hermittian according as A is Hermitian and skew- Hermitian. [5]

3. Find the rank of the matrix $\begin{bmatrix} 6 & 1 & 3 & 8 \\ 4 & 2 & 6 & -1 \\ 10 & 3 & 9 & 7 \\ 16 & 4 & 12 & 15 \end{bmatrix}$ reducing this into the triangular form. [5]

4. Obtain the characteristic equation of the matrix $A = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix}$ and verify that it is satisfied by A. [5]

5. Evaluate $\int_c \vec{F}.\vec{dr}$, where $\vec{F} = (x-y)\,\vec{i} + (x+y)\,\vec{j}$ along the closed curve C bounded by $y^2 = x$ and $x^2 = y$ [5]

6. Find the value of the normal surface integral $\iint_S \vec{F}.\vec{n}\,ds$ for $\vec{F} = x\,\vec{i} - y\,\vec{j} + (z^2-1)\,\vec{k}$, where S is the surface bounded by the cylinder $x^2+y^2=4$ between the planes Z = 0 and Z = 1. [5]

7. Using Green's theorem, find the area of the astroid $x^{\frac{2}{3}} + y^{\frac{2}{3}} = a^{\frac{2}{3}}$ [5]

8. Verify stoke's theorem for $\vec{F} = 2y\,\vec{i} + 3x\,\vec{j} - z^2\,\vec{k}$ where S is the upper half of the sphere $x^2+y^2+z^2=9$ and C is its boundary. [5]

OR

Evaluate the volume intergral $\iiint_V \vec{F}\,dv$, where V is the region bounded by the surface $x=0, y=0, y=6, z=x^2, z=4$ and $\vec{F} = 2xz\,\vec{i} - x\,\vec{j} + y^2\,\vec{k}$

9. Find the Laplace transforms of the following functions [2.5×2]
   a) $t\,e^{-4t}\sin 3t$
   b) $\dfrac{\cos at - \cos bt}{t}$

10. State and prove the second shifting theorem of the Laplace transform. [5]

11. Solve the following differential equation using Laplace transform. [5]

$$\frac{d^2y}{dx^2}+\frac{dy}{dx}-2y=x \text{ given } y(0)=1,\ y'(0)=0$$

12. Obtain the Fourier series for $f(x) = x^2$ in the interval $-\pi < x < \pi$ and hence show that $\sum\frac{1}{n^2}=\frac{1}{1^2}+\frac{1}{2^2}+\frac{1}{3^2}+........=\frac{\pi^2}{6}$ [5]

13. Express $f(x) = x$ as a half-range sine series in $0 < x < 2$ [5]

14. Maximize $Z = 4x_1+5x_2$ subject to constraints [5]

$2x_1 + 5x_2 \leq 25$
$6x_1 + 5x_2 \leq 45$
$x_1 \geq 0$ and $x_2 \geq 0$

graphically

15. Solve the following linear programming problem using the simplex method. [10]

Maximize $P = 50x_1+80x_2$
Subject to $x_1+ 2x_2 \leq 32$
$3x_1 + 4x_2 \leq 84$
$x_1, x_2 \geq 0$

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02 TRIBHUVAN UNIVERSITY

INSTITUTE OF ENGINEERING

Examination Control Division

2068 Chaitra

Exam.	Regular		
Level	BE	Full Marks	80
Programme	BCE, BEL, BEX, BCT, BME, BIE, B. AGRI.	Pass Marks	32
Year / Part	II / I	Time	3 hrs.

Subject: - Engineering Mathematics III *(SH 501)*

- ✓ Candidates are required to give their answers in their own words as far as practicable.
- ✓ *Attempt **All** questions.*
- ✓ *The figures in the margin indicate **Full Marks**.*
- ✓ *Assume suitable data if necessary.*

1. Prove that: $\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}^2 = \begin{vmatrix} 2bc-a^2 & c^2 & b^2 \\ c^2 & 2ac-b^2 & a^2 \\ b^2 & a^2 & 2ab-c^2 \end{vmatrix} = (a^3+b^3+c^3-3abc)^2$. [5]

2. Define Hermition and Skew Hermition matrix. Show that every square matrix can be uniquely expressed as the sum of a Hermition and a skew Hermition. [5]

3. For what value of λ the equation $x + y + z = 1$, $x + 4y + 10z = \lambda^2$ and $x + 2y + 4z = \lambda$ have a solution? Solve them completely in each case. [5]

4. Find the eigen values and eigen vectors of $A = \begin{vmatrix} 3 & -4 & 4 \\ 1 & -2 & 4 \\ 1 & -1 & 3 \end{vmatrix}$. [5]

5. Evaluate $\int_C \vec{F}.\vec{dr}$, Where C: $x^2 = y$ and $y^2 = x$ and $\vec{F} = (x-y)\vec{i} + (x+y)\vec{j}$. [5]

6. State and prove Green theorem in a plane. [5]

7. Verify Guess divergence theorem for $\vec{F} = x^2\vec{i} + 3\vec{j} + yz\vec{k}$. Taken over the cube bounded by $x = 0, x = 1, y = 0, y = 1, z = 0, z = 1$. [5]

8. Find the Laplace transform of the given function (i) $t^2\sin t$ (ii) cosat sinhat. [5]

9. Evaluate $\iint_s \vec{F}.\hat{n}ds$ where $\vec{F} = 3\vec{i} + x\vec{j} - yz\vec{k}$ and s is the surface of the cylinder $x^2 + y^2 = 9$ included in the first octant between the plane $z = 0, z = 4$. [5]

10. Find the inverse Laplace transform: (a) $\dfrac{1}{(S-2)(S+4)}$ (b) $\log\left(\dfrac{s^2+a^2}{s^2}\right)$ [5]

11. Solve the equation using Laplace transform $y'' + 4y' + 3y = t$, $t>0$ $y(0) = 0$, $y'(0) = 1$. [5]

13

12. Obtain a Fourier series to represent the function f(x) = /x/ for $-\pi \le x \le \pi$ and hence deduce $\frac{\pi^2}{8} = \frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \ldots\ldots$ [5]

13. Obtain the half Range Sine Series f(x) = ex in 0<x<1. [5]

OR

Obtain the Fourier series for $f(x) = x - x^2$ where $-1 < x < 1$ as a Fourier series of period 2.

14. Solve the following by using the simplex method: [7.5]

Maximize $P = 15x_1 + 10x_2$,
Subject to
$2x_1 + x_2 \le 10$,
$x_1 + 3x_2 \le 10$,
$x_1, x_2 \ge 0$.

15. Solve by using the dual method: [7.5]

Minimize $C = 21x_1 + 50x_2$,
Subject to $2x_1 + 5x_2 \le 12$,
$3x_1 + 7x_2 \le 17$,
$x_1, x_2 \ge 0$.

OR

Solve the following LPP by using the big M-method:

Maximize $P = 2x_1 + x_2$,
Subject to
$x_1 + x_2 \le 10$,
$-x_1 + x_2 \ge 2$,
$x_1, x_2 \ge 0$.

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02 TRIBHUVAN UNIVERSITY

INSTITUTE OF ENGINEERING

Examination Control Division

2068 Baishakh

Exam.	Regular / Back		
Level	BE	Full Marks	80
Programme	All (Except B.Arch.)	Pass Marks	32
Year / Part	II / I	Time	3 hrs.

Subject: - Engineering Mathematics III

- ✓ Candidates are required to give their answers in their own words as far as practicable.
- ✓ *Attempt **All** questions.*
- ✓ *The figures in the margin indicate **Full Marks**.*
- ✓ *Assume suitable data if necessary.*

1. Using the properties of determinant prove that: [5]

$$\begin{vmatrix} x & 1 & y & 1 \\ 1 & y & 1 & x \\ 1 & x & 1 & y \\ y & 1 & x & 1 \end{vmatrix} = (x+y+2)(x-y)^2(x+y-2)$$

2. If A and B are two non singular matrices of the same order, prove that $(AB)^{-1} = B^{-1}.A^{-1}$. [5]

3. Find the rank of the following matrix reducing to normal form $\begin{bmatrix} 1 & 2 & 1 & 2 \\ 1 & 3 & 2 & 2 \\ 2 & 4 & 3 & 4 \\ 3 & 7 & 4 & 6 \end{bmatrix}$. [5]

4. Find the eigen values and eigen vectors of the matrix $\begin{bmatrix} 2 & -2 & 2 \\ 1 & 1 & 1 \\ 1 & 3 & -1 \end{bmatrix}$. [5]

5. Find the Laplace transform of the following functions: [5]

   a) $te^{-3t}\cos 2t$ b) $\dfrac{e^{at} - \cos 6t}{t}$

6. Find the inverse Laplace transform of the following functions: [5]

   a) $\dfrac{1}{(s-2)(s+2)^2}$ b) $\dfrac{1}{s^2(s+2)}$

7. Solve using Laplace transform $(D^2 + 4D + 3)x = e^{-t}$, where $x(0) = x'(0) = 1$. [5]

8. Obtain a Fourier series for $f(x) = x^3$ in the interval $-\pi \le x \le \pi$. [5]

9. Find the half range sine series for the function $f(x) = x - x^2$ in the interval $0 < x < 1$. [5]

10. Maximize $Z = x_1 + 1.5x_2$ subject to constraints [5]

   $2x_1 + 2x_2 \le 160$

   $x_1 + 2x_2 \le 120$

   $4x_1 + 2x_2 \le 280$

   $x_1 \ge 0$ and $x_2 \ge 0$ graphically.

11. Solve the following linear programming problems by simplex method [10]

Maximize $Z = 15x_1 + 10x_2$
Subject to $2x_1 + 2x_2 \leq 10$
$x_1 + 3x_2 \leq 10$ and $x_1, x_2 \geq 0$

12. Show that the vector field $\bar{F} = (x^2 - yz)\hat{i} + (y^2 - zx)\hat{j} + (z^2 - xy)\hat{k}$ is irrotational. Find the sector function $\varphi(x, y, z)$ such that $\bar{F} = \nabla\varphi$. [5]

13. If S be the part of the surface $Z = 9 - x^2 - y^2$ with $Z \geq 0$ and $\bar{F} = 3x\bar{i} + 3y\bar{j} + Z\bar{k}$, find the flux of F through S. [5]

14. State and prove that Green's theorem in the plane. [5]

15. Evaluate by Stoke's theorem: [5]

$$\int_c (e^x dx + 2y dy - dz)$$

Where c is the curve: $x^2 + y^2 = 4$, $z = 2$.

OR

Verify Gauss divergence theorem for the vector function $\bar{F} = x^2\hat{i} + z\hat{j} + yz\hat{k}$, taken over the unit cube bounded by the planes: $x = 0, x = 1, y = 0, y = 1, z = 0, z = 1$.

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05 TRIBHUVAN UNIVERSITY
INSTITUTE OF ENGINEERING
Examination Control Division
2067 Ashadh

Exam.	Regular/Back		
Level	BE	Full Marks	80
Programme	All (Except B.Arch.)	Pass Marks	32
Year / Part	II / I	Time	3 hrs.

Subject: - Mathematics III

- ✓ Candidates are required to give their answers in their own words as far as practicable.
- ✓ *Attempt **All** questions.*
- ✓ ***All** questions carry equal marks.*
- ✓ *Assume suitable data if necessary.*

1. Using the properties of determinant prove:

$$\begin{vmatrix} a^2+1 & ba & ca & da \\ ab & b^2+1 & cb & db \\ ac & bc & c^2+1 & dc \\ ad & bd & cd & d^2+1 \end{vmatrix} = a^2+b^2+c^2+d^2+1$$

2. Show that every square matrix can be uniquely expressed as the sum of hermitian and a skew-hermitian matrix.

3. Reduce to normal form and find the rank of the matrix:

$$\begin{bmatrix} 2 & -2 & 0 & 6 \\ 4 & 2 & 0 & 2 \\ 1 & -1 & 0 & 3 \\ 1 & -2 & 1 & 2 \end{bmatrix}$$

4. Find the eigen values and eigne vectors of the matrix

$$\begin{bmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{bmatrix}$$

5. Find the Laplace transform of:

   a) coshat sin at   b) $\frac{\cos 2t - \cos 3t}{t}$

6. Find the inverse Laplace transform of:

   a) $\frac{1}{s^2(s^2+a^2)}$ b) $\log\frac{s+1}{s-1}$

7. State and prove the integral theorem of the Laplace transform.

8. Solve the following differential equation using the Laplace transform.

   $y''' + 2y'' - y' - 2y = 0$ where $y(0) = y'(0) = 0$ and $y''(0) = 6$

9. Find a Fourier series to represent $x - x^2$ from x [illegible]. Hence show that

$$\frac{\pi^2}{12} = \frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \ldots$$

10. Express $f(x) = x$ as a cosine half range series in $0 < x < 2$.

11. The acceleration of a moving particle at any time t is given by $\frac{d^2\vec{r}}{dt^2} = 12\cos 2t\hat{i} - 8\sin 2t\hat{j} + 16t\hat{k}$. Find the velocity $\vec{v}$ and displacement $\vec{r}$ at anytime t if

$t = 0$, $\vec{v} = 0$ and $\vec{r} = 0$.

12. Find the angle between the normals to the surface $xy = z^2$ at the points (1,4,2) and (–3,–3,3)

13. Find the work done in moving a particle once round the circle $x^2 + y^2 = 9$, $z = 0$ under the force field $\vec{F}$ given by $\vec{F} = (2x - y + z)\,\vec{i} + (x + y - z^2)\,\vec{j} + (3x - 2y + 4z)\,\vec{k}$.

14. Evaluate $\iint_S \vec{F}.\vec{n}\,ds$ where s is the upper side of triangle with vertices (1,0,0), (0,1,0), (0,0,1) where $\vec{F} =$ ~~$(x - 2z)\,\vec{i}$~~ $+ (x + 3y + z)\,\vec{j} + (5x + y)\,\vec{k}$.

15. State Green's theorem in a plane. Using Green's theorem find the area of $x^{2/3} + y^{2/3} = a^{2/3}$.

16. Verify Stoke's theorem for $\vec{F} = (2x - y)\,\vec{i} - yz^2\,\vec{j} - y^2z\,\vec{k}$ where s is the upper part of the sphere $x^2 + y^2 + z^2 = a^2$ and c is its boundary.

OR

Verify Gauss theorem for $\vec{F} = y\,\vec{i} + x\,\vec{j} + z^2\,\vec{k}$ over the region bounded by $x^2 + y^2 = a^2$, $z = 0$ and $z = h$.

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04 TRIBHUVAN UNIVERSITY
INSTITUTE OF ENGINEERING
Examination Control Division
2067 Magh

Exam.	Back		
Level	BE	Full Marks	80
Programme	All (Except B.Arch.)	Pass Marks	32
Year / Part	II / I	Time	3 hrs.

Subject: - Mathematics III

- ✓ Candidates are required to give their answers in their own words as far as practicable.
- ✓ *Attempt __All__ questions.*
- ✓ *__All__ questions carry equal marks.*
- ✓ *Assume suitable data if necessary.*

1. Show that $\begin{vmatrix} a & b & b & b \\ a & b & a & a \\ a & a & b & a \\ b & b & b & a \end{vmatrix} = -(b-a)^4$.

2. If P and Q are two orthogonal matrices of the same order, prove that their product is also orthogonal.

3. Reducing to normal form, find the rank of matrix $\begin{vmatrix} 1 & -1 & 2 & -3 \\ 4 & 1 & 0 & 2 \\ 0 & 3 & 0 & 4 \\ 0 & 1 & 0 & 2 \end{vmatrix}$.

4. Find the eigen values and eigen vectors of the matrix $\begin{vmatrix} 2 & -2 & 2 \\ 1 & 1 & 1 \\ 1 & 3 & -1 \end{vmatrix}$.

5. Find a Fourier series for $f(x) = x^3, -\pi < x < \pi$.

6. Find the half range sine series for the function $f(x) = e^x$ for $0 < x < \pi$.

7. Find the Laplace transform of

   a) $t^2 \cos at$

   b) $t^3 e^{-3t}$

8. Find the Inverse Laplace transform of

   a) $\dfrac{s}{(s-3)(s^2+4)}$

   b) $\log \dfrac{s(s+1)}{(s^2+4)}$

9. If $L\{f(t)\} = F(s)$, then prove $L\{e^{at} f(t)\} = F(s-a)$.

10. Use the Laplace transform to solve $\frac{d^2y}{dt^2}+4\frac{dy}{dt}+3y=e^{-t}$, $y(0)=y^1(0)=1$.

11. The position vector of a moving particle at any time t is given by $\vec{r}=(t^2+1)\vec{i}+(4t-3)\vec{j}+(2t^2-6)\vec{k}$. Find the velocity and acceleration at t = 1. Also find their magnitudes.

12. Define divergence and curl of $\vec{V}$. Prove that div(Curl $\vec{V}$) = 0.

13. Evaluate $\int_c \vec{F}.\vec{dr}$ where $\vec{F}=Z\vec{i}+x\vec{j}+y\vec{k}$ and C is the arc of curve, $x=t^2+1$, $y=2t^2$, $z=t^3$ from t = 1 to t = 2.

14. Evaluate $\iint_S \vec{F}.\vec{n}\,ds$ where $\vec{F}=x\vec{i}+y\vec{j}+z\vec{k}$ and S is the outside of the lateral surface of circular cylinder, $x^2+y^2=a^2$ between planes z = 0 and z = 4.

15. Use Green's theorem to find the area of ellipse, $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$.

16. Verify Stoke's theorem for $\vec{F}=x\vec{i}+z^2\vec{j}+y^2\vec{k}$ over the plane surface x + y + z = 1 lying in first octant.

OR

Verify Gauss's theorem for $\vec{F}=4x\vec{i}-2y^2\vec{j}+z^2\vec{k}$ taken over the region bounded by $x^2+y^2=4$, z =0 and z =3.

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